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Tests for Serial Dependence and Other Specification Analysis in Models of Markets in Disequilibrium

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Tests for Serial Dependence and Other Specification Analysis in Models of Markets in Disequilibrium

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Abstract

The assumption of serial independence of the disturbance terms is the starting point of almost all the work that has been done on analyzing market disequilibrium models. Under serial dependence the usual maximum likelihood estimator (MLE) will be inefficient although they may remain consistent. Two other assumptions about the disturbance terms, namely normality and homoscedasticity, are also usually made. Violation of these is likely to make the MLE to be inconsistent. In this paper, we first derive tests for serial dependence given normality and homoscedasticity using the Lagrange multiplier (LM) test principle. The likelihood function under serial dependence is very complicated and involves multiple integrals of dimensions equal to the sample size, ruling out the possibility of using the Wald and likelihood ratio tests. However, the test statistic we obtain through the LM principle is very simple. Next, we relax the normality assumption using the Box-Cox transformation family and generalize the above test to such non-normal cases. We also suggest a general specification test for the disequilibrium market model, namely White's information matrix test, which can detect non-normality, heteroscedasticity and serial dependence jointly. This procedure will be useful when there is not much information about the alternative hypothesis. In the last part of the paper we present an analysis of disequilibrium models assuming that the disturbances are logistic rather than normal. This modification does simplify the computations quite a lot since for the logistic distribution a simple closed form expression for its distribution function is available. The relative performance of this distribution compared to the normal distribution is also investigated.

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1. Introduction

Models of markets in disequilibrium (MMD) have been studied, both theoretically and empirically, by many authors in recent years [see Quandt (1982) or Maddala (1983), for a review]. Unlike many other econometric models, however, MMD have yet to be subjected to rigorous specification analysis [though see Quandt (1981) and Lee (1982, 1984)]. In most studies, for example, disturbances are assumed to be serially independent (I), whereas MMD are designed to describe time series data. Since autocorrelation in economic time series is not always accounted for by the exogenous variables, serial dependence might be expected to be present. Fair and Jaffee (1972) incorporated serial correlation in their model, but for estimation they had to make an assumption that there was an equilibrium (that is, both the demand and supply were equal to the actual quantity transacted) before every switching point. Fair and Kelejian (1974) noted that when the disturbances are serially correlated, the coefficients in the demand and supply equations are not identified. Amemiya (1974) and Maddala and Nelson (1974) commented that serial correlation is a serious problem and, when incorporated into the model, it renders maximum likelihood estimation computationally intractable. The usual maximum likelihood estimators (MLEs) maximize a likelihood based on serially independent, identically distributed, normal disturbances. We continue to call these MLEs even when one or more of the assumptions are in fact breached by the data to hand. Presence of serial dependence ($\overline{I}$) will render the MLEs asymptotically inefficient, though they may still be consistent. Failure of the other assumptions, normality (N) and homoscedasticity (H), may lead to inconsistency of the MLEs.

In the present paper, we develop some specification tests, with an emphasis on $\bar{I}$, for a particular type of disequilibrium model, namely Model 1 of Maddala and Nelson (1974). Other MMD, such as Model 2 of Maddala and Nelson, which assume knowledge of sample separation will be easier to handle. Throughout, the Lagrange multiplier (LM) or score test principle is adopted, partly because of its good asymptotic power properties but mostly because the likelihood under I is vastly more tractable than that under $\bar{I}$. As noted by Maddala and Nelson (1974), the latter involves multiple integrals of dimension equal to the sample size. [Quandt (1981) gives an expression under first-order autoregressive (AR(1)) disturbances, that involves only univariate integrals but, as noted by Lee (1984), and as we indicate in the next section, this is not the likelihood.] Thus Wald and likelihood ratio tests are not feasible.

In the next section we derive the LM test for $\bar{I}$ assuming NH, our analysis being along the lines of Robinson et al. (1985) where a similar test for the Tobit model was proposed. The class of possible alternatives to I is, in certain respects, broader than those in Lee (1984) where he considered only AR(1) processes, but Lee's maintained assumption of contemporaneously correlated residuals is more general than ours. In another paper Lee (1982) also derived a test for non-normality ($\bar{N}$) with alternatives belonging to the bivariate Edgeworth series of distributions. His test strongly rejected the normality assumption when applied to the housing data of Fair and Jaffee (1972). The tests suggested in Section 2 will not be valid under $\bar{N}$. Therefore, in Section 3, we relax the normality assumption, using the Box-Cox transfor-

mation, and test for $\overline{I}$ given $\overline{NH}$. Subsequently, the possibility of heteroscedasticity ($\overline{H}$) is also introduced. Then using White (1982) information matrix test principle, we suggest a general specification test. This procedure can be viewed as a simultaneous test of $\overline{INH}$. Such a test is not likely to be very powerful when only one or two of I, N or H are violated, but it does seem useful as a first step when there is little information concerning possible departures from the classical assumptions. A strategy to be adopted should any of the tests reject would be desirable, as would be a development of tests of $\overline{N}$ and $\overline{H}$ given $\overline{I}$. Unfortunately both objectives are hampered by the difficulty, as previously mentioned, of ML estimation under $\overline{I}$. It may be worth investigating whether at least consistent estimation of serial correlation is possible, for example by moment methods or methods similar to those of Robinson (1982b), which apply to the Tobit model.

It is now well known that normality of the disturbances is not an assumption that can be taken for granted in the context of MMD; nor, unlike in the usual regression model, does it lead to neat statistical properties or to computational convenience. It is, therefore, worth considering alternative distributions which may be just as relevant as the normal but make computation easier. Goldfeld and Quandt (1981) considered a Laplace type distribution which they called the Sargan density. However, for the simple disequilibrium model this did not lead to much computational gain, perhaps due to the "piecewise" nature of their likelihood function. In our view, in this situation, a better alternative is the logistic distribution. The shape of the logistic distribution is very similar to the normal, with a slightly

thicker tail, and, most important, it has a closed from expression for its distribution function. In Section 4, we study the likelihood function assuming a logistic distribution, and generate data under the normal, logistic and Student's t distributions and make a comparative study of the performances of the normal and logistic distributions.

2. Tests for Serial Dependence under Normality

2.1 Likelihood Function with Serial Dependence

We consider the following simple disequilibrium model

$$D_t = x_{1t}'\beta_1 + u_{1t}, \tag{2.1}$$

$$S_t = x_{2t}'\beta_2 + u_{2t}, \tag{2.2}$$

$$Q_t = \min(D_t, S_t) \quad t=1,2,\ldots, \tag{2.3}$$

where D_t, S_t and Q_t denote respectively the quantity demanded, supplied and transcated at period t, x_{1t} and x_{2t} are column vectors of k_1 and k_2 exogenous variables and u_{1t} and u_{2t} are the disturbance terms such that $E(u_{it}|x_{is}, s=1,2,\ldots) = 0$, $i = 1,2$. We also assume, like Quandt (1981), but unlike Lee (1984), that u_{1t} and u_{2s} are independent for all t and s.

Quandt (1981) assumed that the u_{it} are also AR(1) and set up the function

$$\prod_{t=1}^{T} h_t(Q_t|D_o,S_o), \tag{2.4}$$

where $h_t(Q_t|D_o,S_o)$ is the conditional density of Q_t given D_o,S_o, and involves only univariate integrals. Quandt (1981) described (2.4) as the likelihood function, but in fact the latter is of the form

$$h(Q_1,Q_2,\ldots Q_T) = h_1(Q_1) \prod_{t=2}^{T} h_t(Q_t|Q_{t-1},\ldots,Q_1). \tag{2.5}$$

Although estimates maximizing (2.4) may be consistent, they are not efficient, and indeed the autocorrelation coefficients seem likely to be poorly identified by (2.4) because $h_t(Q_t|D_o,S_o) \to 0$ as $t \to \infty$. Perhaps this explains the seeming contradiction between the significance of the likelihood ratio test for $\bar{I}$ and the insignificance of the individual autocorrelation coefficients, mentioned in Quandt (1981), though this might as well be due to the presence of other misspecification(s) such as $\bar{N}$ or a wrong functional form in (2.1) and/or (2.2).

Let $f(D,S;\theta)$ be the joint probability density function (p.d.f) of the 1x2T vector $(D,S) = (D_1,D_2,\ldots,D_T,S_1,S_2,\ldots,S_T)$, which is a given function of D, S and an unknown parameter vector θ. The log-likelihood function is

$$L(\theta) = \log \sum_m \int_R f(D,S;\theta) \prod_{t=1}^{T} (dD_t)^{w_{mt}} (dS_t)^{1-w_{mt}} \Big|_Q , \tag{2.6}$$

where

$$Q = (Q_1,Q_2,\ldots,Q_T), \quad \int_R = \int_{Q_1}^{\infty} \cdots \int_{Q_T}^{\infty},$$

$$w_{mt} = 1 \text{ if } Q_t = D_t,$$

$$= 0 \text{ if } Q_t = S_t,$$

and the sum is over all 2^T choices of $(w_{m1},w_{m2},\ldots,w_{mT})$, that is, over all possible demand/supply sequences. Under our setup D and S are independent, i.e.,

$$f(D,S;\theta) = f_1(D;\theta_1)f_2(S;\theta_2), \tag{2.7}$$

where $\theta' = (\theta_1', \theta_2')$.

We now introduce $\bar{I}$ explicitly into the model. As a class of alternatives to independence we take, as in Robinson (1982a), u_{it} to be generated by a stationary Gaussian process. The j-th autocorrelation of u_{it}, $\rho_{ij}(\psi_i)$, is a uniquely defined, differentiable function of a p_i-dimensional column vector ψ_i, i=1,2. Let $R_i(\psi_i)$ be the TxT Toeplitz matrix with (j,j+k)-th element $\rho_{ik}(\psi_i)$. If $\phi_T(y;\mu,\Omega)$ denotes the p.d.f of a T-variate normal distribution with mean μ and variance-covariance matrix Ω, the log-likelihood function can be written as

$$L(\theta) = \log \sum_m \int_R \phi_T(D;X_1\beta_1,\sigma_1^2R_1(\psi_1)) \cdot \phi_T(S;X_2\beta_2,\sigma_2^2R_2(\psi_2))$$

$$\prod_{t=1}^{T} (dD_t)^{w_{mt}} (dS_t)^{1-w_{mt}} \Big|_Q , \tag{2.8}$$

where $\sigma_i^2 = V(u_{it})$ and X_i is the Txk$_i$ matrix with t-th row x_{it}, i=1,2.

2.2 The Test Statistics

We assume, as in Robinson et al. (1985), for a unique value of ψ_i, which we take with no loss of generality to be the vector of zeros, that $\rho_{ij}(0) = 0$, i=1,2; j=1,2,... . So a test of $\bar{I}$ can be expressed as

$$H_o: \ \psi_i = 0, \ i = 1,2 \text{ against } H_1: \ \psi_i \neq 0, \text{ for some } i. \tag{2.9}$$

The alternatives we shall emphasize are the p_i-th order autoregression [AR(p_i)] and the p_i-th order moving average [MA(p_i)] processes. As in

the classical regression case, the same LM statistic will result from both AR(p_i) and MA(p_i) alternatives. The statistic falls out quickly from the general form which we shall derive. Statistics for various other types of alternative can be readily obtained from our general form.

It is clear from the expression (2.8) that the likelihood function involves normal integrals of dimension equal to the sample size, and in general it cannot be expressed in terms of normal integrals of unit dimension. Therefore, maximization of (2.8) will be hampered by the need to compute integrals of very large dimension. Thus the Wald and likelihood ratio tests are not at all practically feasible. It can be shown that under H_o, when $R_1(\psi_1) = R_2(\psi_2) = I_T$, where I_T is an identity matrix of dimension T, (2.8) reduces to the log-likelihood function given in Maddala and Nelson (1974, p. 1015). To see that and for our future use, we introduce the following notations:

$$\theta_i' = (\beta_i', \sigma_i^2, \psi_i'), \quad \theta_{io}' = (\beta_i', \sigma_i^2, 0'), \quad \theta_o' = (\theta_{1o}', \theta_{2o}'),$$

$$\theta_A' = (\beta_1', \sigma_1^2, \beta_2', \sigma_2^2), \quad \theta_B' = (\psi_1', \psi_2'), \quad \theta' = (\theta_A', \theta_B'),$$

$$f_i(Q_t) = \phi_1(Q_t; x_{it}'\beta_i, \sigma_i^2), \quad i = 1,2,$$

$$F_1(Q_t) = \int_{Q_t}^{\infty} \phi_1(D_t; x_{1t}'\beta_1, \sigma_1^2)dD_t,$$

and

$$F_2(Q_t) = \int_{Q_t}^{\infty} \phi_1(S_t; x_{2t}'\beta_2, \sigma_2^2)dS_t.$$

For convenience, we will drop the symbol Q and write $f_i(Q_t)$ as f_{it} etc. Now from (2.8)

$$L(\theta_o) = \log \sum_m \int_R \prod_{t=1}^{T} \{f_1(D_t)f_2(S_t)(dD_t)^{w_{mt}}(dS_t)^{1-w_{mt}}\}\Big|_Q$$

$$= \log \sum_m \prod_{t=1}^{T} \{w_{mt}F_{1t}f_{2t} + (1-w_{mt})f_{1t}F_{2t}\}$$

$$= \sum_{t=1}^{T} \log \{F_{1t}f_{2t} + f_{1t}F_{2t}\} . \qquad (2.10)$$

Since the log-likelihood function simplifies considerably under H_o, we turn to the LM approach and develop tests for $\bar{I}$.

Let us consider the partition $\theta' = (\theta_A', \theta_B')$ where we want to test H_o: $\theta_B = 0$. There are many asymptotically equivalent forms of the LM test statistic and we will use the following form:

$$LM = \frac{\partial L(\tilde{\theta})'}{\partial \theta_B} \tilde{M}^{-1} \frac{\partial L(\tilde{\theta})}{\partial \theta_B} ,$$

where "$\tilde{}$" indicates the quantities have been evaluated at the restricted MLE of θ say $\tilde{\theta}$ and M satisfies

$$\operatorname*{plim}_{T\to\infty} \left[\frac{1}{T} E \left\{- \frac{\partial^2 L(\theta)}{\partial \theta_B \partial \theta_B'}\right\}\Big|_{\tilde{\theta}} - M\right] = 0.$$

The above form is valid since under our setup

$$E_{H_o} \left\{\frac{\partial^2 L(\theta)}{\partial \theta_A \partial \theta_B'}\right\} = 0 \qquad (2.11)$$

as shown in the subsequent derivation.

To derive LM we require the derivatives (under H_o)

$$\left.\begin{aligned}\frac{\partial}{\partial\psi_{1j}}\log\phi_T(D;X_1\beta_1,\sigma_1^2R_1(\psi_1)) &= \frac{1}{2\sigma_1^2}(D-X_1\beta_1)'R_{1j}(D-X_1\beta_1),\\ \frac{\partial}{\partial\psi_{2j}}\log\phi_T(S;X_2\beta_2,\sigma_2^2R_2(\psi_2)) &= \frac{1}{2\sigma_2^2}(S-X_2\beta_2)'R_{2j}(S-X_2\beta_2),\end{aligned}\right\} \qquad (2.12)$$

where $R_{ij} = (\partial/\partial\psi_{ij})R_i(0)$, for $j = 1,2,\ldots,p_i$.

Denoting $\rho_{ikj} = (\partial/\partial\psi_{ij})\rho_{ik}(0)$ and using (2.12) we have

$$\frac{\partial}{\partial\psi_{1j}}[\exp L(\theta_o)] = \frac{1}{2\sigma_1^2}\sum_{\substack{t,u=1\\ t\neq u}}^{T}\rho_{1,t-u,j}\sum_m\int_R(D_t-x_{1t}'\beta_1)(D_u-x_{1u}'\beta_1)$$

$$\cdot\prod_{n=1}^{T}\{f_1(D_n)f_2(S_n)(dD_n)^{w_{mn}}(dS_n)^{1-w_{mn}}\}\Big|_Q .$$

Now

$$\int_{Q_t}^{\infty}(D_t-x_{1t}'\beta_1)f_1(D_t)f_2(S_t)(dD_t)^{w_{mt}}(dS_t)^{1-w_{mt}}\Big|_{Q_t}$$

$$= w_{mt}f_2(Q_t)\int_{Q_t-x_{1t}'\beta_1}^{\infty} z_t\phi_1(z_t;0,\sigma_1^2)dz_t$$

$$+ (1-w_{mt})(Q_t-x_{1t}'\beta_1)\cdot f_1(Q_t)\int_{Q_t-x_{2t}'\beta_2}^{\infty}\phi_1(z_t;0,\sigma_2^2)dz_t$$

$$= w_{mt}\sigma_1^2f_{1t}f_{2t} + (1-w_{mt})(Q_t-x_{1t}'\beta_1)f_{1t}F_{2t} .$$

Thus

$$\frac{\partial}{\partial \psi_{1j}} [\exp L(\theta_o)] = \frac{1}{2\sigma_1^2} \sum_{\substack{t,u=1 \\ t\neq u}}^{T} \rho_{1,t-u,j}$$

$$\cdot \sum_{m} \{w_{mt}\sigma_1^2 f_{1t} f_{2t} + (1-w_{mt})(Q_t - x_{1t}'\beta_1) f_{1t} F_{2t}\}$$

$$\cdot \{w_{mu}\sigma_1^2 f_{1u} f_{2u} + (1-w_{mu})(Q_u - x_{1u}'\beta_1) f_{1u} F_{2u}\}$$

$$\cdot \prod_{k\neq t,u}^{T} \{w_{mk} F_{1k} f_{2k} + (1-w_{mk}) f_{1k} F_{2k}\}.$$

Following our derivation of (2.10), we can write

$$\frac{\partial}{\partial \psi_{1j}} [\exp L(\theta_o)] = \frac{1}{2\sigma_1^2} \sum_{\substack{t,u=1 \\ t\neq u}}^{T} \rho_{1,t-u,j} \gamma_{1t} \gamma_{1u}$$

$$\cdot \prod_{\substack{k=1 \\ k\neq t,u}}^{T} \{F_{1k} f_{2k} + f_{1k} F_{2k}\},$$

where $\gamma_{1t} = f_{1t}[\sigma_1^2 f_{2t} + (Q_t - x_{1t}'\beta_1)F_{2t}]$. Therefore,

$$\frac{\partial}{\partial \psi_{1j}} [L(\theta_o)] = \frac{1}{2\sigma_1^2} \frac{\sum_{\substack{t,u=1 \\ t\neq u}}^{T} \rho_{1,t-u,j} \gamma_{1t} \gamma_{1u} \prod_{\substack{k=1 \\ k\neq t,u}}^{T} (F_{1k} f_{2k} + f_{1k} F_{2k})}{\prod_{k=1}^{T} (F_{1k} f_{2k} + f_{1k} F_{2k})}$$

$$= \frac{1}{\sigma_1^2} \sum_{k=1}^{T-1} \rho_{1kj} \sum_{t=k+1}^{T} \nu_{1t} \nu_{1,t-k},$$

where

$$\nu_{1t} = \frac{\gamma_{1t}}{F_{1t}f_{2t} + f_{1t}F_{2t}} = \frac{\sigma_1^2 f_{1t}f_{2t} + (Q_t - x_{1t}'\beta_1)f_{1t}F_{2t}}{F_{1t}f_{2t} + f_{1t}F_{2t}} .$$

Under H_o, the ν_{1t} are independent, and noting that the denominator of ν_{1t} is the p.d.f of Q_t, say $h(Q_t)$, we have

$$E(\nu_{1t}) = \int_{-\infty}^{\infty} \frac{\gamma_{1t}}{h(Q_t)} \cdot h(Q_t)dQ_t .$$

Since $\frac{d}{dQ_t}(f_{1t}F_{2t}) = -f_{1t}f_{2t} \frac{1}{\sigma_1^2}(Q_t - x_{1t}'\beta_1)f_{1t}F_{2t}$, we can write

$$E(\nu_{1t}) = -\sigma_1^2 \int_{-\infty}^{\infty} \frac{d(f_{1t}F_{2t})}{dQ_t} dQ_t = -\sigma_1^2[f_{1t}F_{2t}]_{-\infty}^{\infty} = 0.$$

Similarly,

$$\frac{\partial}{\partial\psi_{2j}} [L(\theta_o)] = \frac{1}{\sigma_2^2} \sum_{k=1}^{T-1} \rho_{2kj} \sum_{t=k+1}^{T} \nu_{2t}\nu_{2,t-k},$$

where $\nu_{2t} = \frac{\sigma_2^2 f_{1t}f_{2t} + (Q_t - x_{2t}'\beta_2)f_{2t}F_{1t}}{F_{1t}f_{2t} + f_{1t}F_{2t}}$,

with $E(\nu_{2t}) = 0$. Thus

$$\frac{\partial}{\partial\psi_i} [L(\theta_o)] = \frac{1}{\sigma_i^2} \sum_{k=1}^{T-1} \rho_{ik} \sum_{t=k+1}^{T} \nu_{it}\nu_{i,t-k},$$

where

$$\rho_{ik} = \begin{bmatrix} \rho_{ik1} \\ \rho_{ik2} \\ \vdots \\ \rho_{ikp_i} \end{bmatrix}, \quad i=1,2.$$

Next

$$\frac{\partial^2}{\partial\beta_i\partial\psi_i'}[L(\theta_o)] = \frac{1}{\sigma_i^2}\sum_{k=1}^{T-1}\rho_{ik}\sum_{t=k+1}^{T}\{\nu_{it}(\frac{\partial}{\partial\beta_i}\nu_{i,t-k}) + (\frac{\partial}{\partial\beta_i}\nu_{it})\nu_{i,t-k}\},$$

$$\frac{\partial^2}{\partial\sigma_i^2\partial\psi_i'}[L(\theta_o)] = -\frac{1}{\sigma_i^2}\frac{\partial}{\partial\psi_i}L(\theta_0) + \frac{1}{\sigma_i^2}\sum_{k=1}^{T-1}\rho_{ik}\sum_{t=k+1}^{T}\{\nu_{it}(\frac{\partial}{\partial\sigma_i^2}\nu_{i,t-k})$$

$$+ (\frac{\partial}{\partial\sigma_i^2}\nu_{it})\nu_{i,t-k}\},$$

and these have zero expectations, by the independence of ν_{it} over t and $E(\nu_{it}) = 0$. Of course

$$E\{\frac{\partial 2}{\partial\beta_j\partial\psi_i'}[L(\theta_o)]\} = 0, \text{ and } E\{\frac{\partial}{\partial\sigma_j^2\partial\psi_i'}[L(\theta_o)]\} = 0,$$

for $i \neq j$, verifying equation (2.11) and

$$E\{\frac{\partial}{\partial\psi_i\partial\psi_j'}[L(\theta_o)]\} = 0,$$

for $i \neq j$. Finally,

$$E\{\frac{\partial}{\partial\psi_{ij}} L(\theta_o) \frac{\partial}{\partial\psi_{i\ell}} L(\theta_o)\}$$

$$= \frac{1}{\sigma_i^4} E[\sum_{k=1}^{T-1} \sum_{m=1}^{T-1} \rho_{ikj}\rho_{im\ell} \sum_{t=k+1}^{T} \sum_{u=k+1}^{T} \nu_{it}\nu_{i,t-k}\nu_{iu}\nu_{i,u-m}]$$

$$= \frac{1}{\sigma_i^4} \sum_{k=1}^{T-1} \rho_{ikj}\rho_{ik\ell} E[\sum_{t=k+1}^{T} \nu_{it}^2 \nu_{i,t-k}^2]$$

$$= \sum_{k=1}^{T-1} \rho_{ikj}\rho_{ik\ell} \sum_{t=k+1}^{T} \lambda_{it}\lambda_{i,t-k} \text{ where } \lambda_{it} = \frac{1}{\sigma_i^2} E(\nu_{it}^2). \qquad (2.13)$$

A closed form expression for $E(\nu_{it}^2)$ does not appear to be available, however, so we replace (2.13) by

$$M_{ij\ell} = \sum_{k=1}^{T-1} \rho_{ikj}\rho_{ik\ell}c_{ik}, \quad c_{ik} = \frac{1}{\tilde{\sigma}_i^4} \sum_{t=k+1}^{T} \tilde{\nu}_{it}^2 \tilde{\nu}_{i,t-k}^2,$$

where $\tilde{\nu}_{it}$ is ν_{it} with β_i and σ_i^2 replaced by their MLEs $\tilde{\beta}_i$ and $\tilde{\sigma}_i^2$. The LM statistic is thus given by

$$LM = \sum_{i=1}^{2} \ell_i' M_i \ell_i,$$

where $\ell_i = \frac{1}{\tilde{\sigma}_i 2} \sum_{k=1}^{T-1} \rho_{ik} d_{ik}$ with $d_{ik} = \sum_{t=k+1}^{T} \tilde{\nu}_{it}\tilde{\nu}_{i,t-k}$ and $M_i = ((M_{ij\ell}))$.

When the null hypothesis H_o is true, under certain regularity conditions

$$LM \xrightarrow{D} \chi^2_{p_1 + p_2}. \qquad (2.14)$$

These regularity conditions may be established using the results of Hartley and Mallela (1977) and combining the conditions of Robinson et al. (1985) on the $\rho_{ij}(\psi_i)$.

We now deduce the form of LM for testing against $AR(p_i)$ and $MA(p_i)$ alternatives. For both the $AR(p_i)$ and $MA(p_i)$

$$\rho_{ikj} = 1 \text{ for } k = j$$

$$= 0 \text{ for } k \neq j,$$

so that in either case, the LM test statistic reduces to

$$LM(p_1,p_2) = T \sum_{i=1}^{2} \sum_{k=1}^{p_i} r_{ik}^2, \tag{2.15}$$

where $r_{ik} = \frac{1}{\sqrt{T}} \frac{d_{ik}}{\tilde{\sigma}_i^2 c_{ik}^{1/2}}$. Note that (2.15) is analogous to the familiar Box-Pierce portmanteau statistic for testing for serial correlation.

2.3 Empirical Illustration

We apply the test to the housing start model of Fair and Jaffee (1972). Their versions of (2.1)-(2.3) are [see Maddala and Nelson (1974, p. 1025)]

$$D_t = \alpha_0 + \alpha_1 z_{1t} + \alpha_2 z_{2t} + \alpha_3 z_{3t} + u_{1t}, \tag{2.16}$$

$$S_t = \beta_0 + \beta_1 z_{1t} + \beta_2 z_{4t} + \beta_3 z_{5t} + \beta_4 z_{6t} + u_{2t}, \tag{2.17}$$

$$Q_t = \min(D_t, S_t), \tag{2.18}$$

where Q_t is the observed number of housing starts in period t, z_{1t} is a time trend, z_{2t} is a measure of the stock of houses, z_{3t} is the mortgage rate lagged two periods, z_{4t} is the six month moving average of the flow of private deposits in savings and loan associations (SLAs) and mutual savings banks, lagged one period, z_{5t} is the flow of borrowings by the SLAs from the Federal Home Loan Bank lagged two periods, and $z_{6t} = z_{3,t+1}$. Using the Fair and Jaffee (1972) data with a sample of size 126, Maddala and Nelson (1974, Table II) obtained the MLEs which are reproduced below:

Parameter	α_0	α_1	α_2	α_3	β_0	β_1	β_2	β_3	β_4	σ_1^2	σ_2^2
Estimate	223.74	2.52	-.022	-.090	15.55	-.153	.053	.053	.093	350.0	80.2

Lee (1982) found evidence of no contemporaneous correlation between the u_{it} for these data, so the setting for an LM test for $\bar{I}$ seems appropriate here. We use the above estimates and obtain the following components of the LM statistic given in (2.15):

$$Tr_{11}^2 = 4.4733 \qquad Tr_{21}^2 = 23.9101 \qquad Tr_{25}^2 = 6.0109$$

$$Tr_{12}^2 = .4636 \qquad Tr_{22}^2 = 10.3031 \qquad Tr_{26}^2 = .9269$$

$$Tr_{13}^2 = 1.7940 \qquad Tr_{23}^2 = 9.0344 \qquad Tr_{27}^2 = .3906$$

$$Tr_{14}^2 = .0003 \qquad Tr_{24}^2 = 6.7310 \qquad Tr_{28}^2 = .2724$$

It seems, the length of the disturbance AR process is short for the demand equation whereas for the supply equation it is quite the opposite. Using the above values we can form a number of test statistics, e.g.,

LM(1,0) = 4.4733	LM(0,1) = 23.9101
LM(1,1) = 28.3834	LM(1,2) = 38.6865
LM(2,1) = 28.8470	LM(2,2) = 39.1501

The χ^2 critical values for 1, 2, 3, and 4 degrees of freedom at one percent significant level are respectively 6.63, 9.21, 11.34, and 13.28. Therefore, we strongly reject the null hypothesis of no serial dependence. The inference remains the same even if we consider higher values of p_1 and p_2. Here we should note that our sample size is 126, and the asymptotic χ^2 critical values are unlikely to be appropriate. However, for this particular example, the statistic values for the supply equation are so large that the conclusion of u_{2t} being serially correlated seems justified insofar as the maintained assumption of correct specification, normality and homoscedasticity are valid.

3. Tests for Serial Dependence in the Presence of Non-normality and A General Specification Test

3.1 The Test Statistics for Serial Dependence under Non-normality

In the last section, a test was derived under the assumption that the disturbances are normally distributed. When this assumption is violated, the usual MLEs will be inconsistent and the test will be invalid. We now allow for $\bar{N}$ using the Box-Cox transformation family, generalizing our previous test.

We consider the model as given in (2.1)-(2.3) except now assume that initially the u_{it}'s are not normally distributed but a Box-Cox transformation of the dependent variables restores normality. Therefore, now we have

$$D_t^{(\lambda_1)} = x_{1t}'\beta_1 + u_{1t}, \tag{3.1}$$

$$S_t^{(\lambda_2)} = x_{2t}'\beta_2 + u_{2t}, \tag{3.2}$$

$$Q_t = \min(D_t, S_t), \tag{3.3}$$

where

$$\left.\begin{aligned} D_t^{(\lambda_1)} = T(D_t,\lambda_1) &= \frac{D_t^{\lambda_1} - 1}{\lambda_1} \quad \text{if } \lambda_1 \neq 0, \\ &= \log D_t \quad \text{if } \lambda_1 = 0, \end{aligned}\right\} \tag{3.4}$$

and the p.d.f. of $D^{(\lambda_1)}$ is $\phi_T(D^{(\lambda_1)}; X_1\beta_1, \sigma_1^2(R_1(\psi_1)))$. Similarly we define $S_t^{(\lambda_2)}$ and its p.d.f.

Strictly speaking, the u_{it}'s cannot be normal because of the nature of the transformation in (3.1) and (3.2). For (3.4) to be well defined, D_t and S_t should always be positive. This implies the u_{it}'s cannot be normal. An appropriate assumption in this situation would be that the u_{it}'s are truncated normal [see Poirier (1978)]

$$T(0,\lambda_i) - x_{it}'\beta_i < u_{it} < T(\infty,\lambda_i) - x_{it}'\beta_i .$$

However, here we are dealing with macro demand and supply functions; therefore, the $x_{it}'\beta_i$'s are positive and can be expected to be much larger than $|u_{it}|$. Hence, for all practical purposes, we can neglect the effects of truncation.

In this case $\vartheta_A' = (\beta_1', \sigma_1^2, \lambda_1, \beta_2', \sigma_2^2, \lambda_2)$, $\vartheta_B' = (\psi_1', \psi_2')$, $\vartheta' = (\vartheta_A', \vartheta_B')$ and the log-likelihood function for the model (3.1)-(3.3) can be written as

$$L(\theta) = \log \sum_m \int_R \xi_T(D;\ X_1\beta_1,\ \lambda_1,\ \sigma_1^2 R_1(\psi_1)) \cdot \xi_T(S;\ X_2\beta_2,\ \lambda_2,\ \sigma_2^2 R_2(\psi_2))$$

$$\prod_{t=1}^{T} (dD_t)^{w_{mt}} (dS_t)^{1-w_{mt}} \Big|_Q ,$$

where

$$\xi_T(D;\ X_1\beta_1,\ \lambda_1,\ \sigma_1^2 R_1(\psi_1)) = (2\pi)^{-T/2} |\sigma_1^2 R_1(\psi_1)|^{-\frac{1}{2}}$$

$$\cdot \exp\left[-\frac{1}{2\sigma_1^2} (D^{(\lambda_1)} - X_1\beta_1)' R_1(\psi_1)^{-1} (D^{(\lambda_1)} - X_1\beta_1)\right] \cdot \prod_{t=1}^{T} D_t^{\lambda_1 - 1} ,$$

and similarly we define $\xi_T(S;\ X_2\beta_2,\ \lambda_2,\ \sigma_2^2 R_2(\psi_2))$.

Now let

$$\theta_i' = (\beta_i',\ \sigma_i^2,\ \lambda_i,\ \psi_i'),\ \theta_{io}' = (\beta_i',\ \sigma_i^2,\ \lambda_i,\ 0'),\ i = 1,2,$$

$$\theta' = (\theta_1',\ \theta_2'),\ \text{and}\ \theta_o' = (\theta_{1o}',\ \theta_{2o}').$$

Then proceeding as in the previous section, we have

$$\frac{\partial}{\partial \psi_{1j}} [\exp L(\theta_o)] = \frac{1}{2\sigma_1^2} \sum_{\substack{t,u=1 \\ t \neq u}}^{T} \rho_{1,t-u,j} \sum_m (D_t^{(\lambda_1)} - x_{1t}'\beta_1)(D_u^{(\lambda_1)} - x_{1u}'\beta_1)$$

$$\cdot \prod_{n=1}^{T} \{f_1(D_n) f_2(S_n) (dD_n)^{w_{mn}} (dS_n)^{1-w_{mn}}\} \Big|_Q ,$$

where $f_1(D_t) = \varphi_1(D_t^{(\lambda_1)};\ x_{1t}'\beta_1, \sigma_1^2) D_t^{\lambda_1 - 1}$. Now

$$\int_{Q_t}^{\infty} (D_t^{(\lambda_1)} - x_{1t}'\beta_1) f_1(D_t) f_2(S_t) (dD_t)^{w_{mt}} (dS_t)^{1-w_{mt}} \Big|_{Q_t}$$

$$= w_{mt}\{\sigma_1^2 \bar{f}_{1t} f_{2t}\} + (1-w_{mt})(Q_t^{(\lambda_1)} - x_{1t}'\beta_1) f_{1t} F_{2t},$$

where $\bar{f}_{1t} = \phi_1(Q_t^{(\lambda_1)}; x_{1t}'\beta_1, 1)$ and $F_{2t} = \int_{Q_t^{(\lambda_2)}}^{\infty} \phi_1(z_t; x_{2t}'\beta_2, \sigma_2^2) dz_t$,

since $\int_{Q_t}^{\infty} (D_t^{(\lambda_1)} - x_{1t}'\beta_1) f_1(D_t) dD_t = \sigma_1^2 \phi_1(Q_t^{(\lambda_1)}; x_{1t}'\beta_1, 1)$.

Therefore, the derivations and the resulting test statistic are essentially the same as in the previous section with new definitions of $f_i(Q_t)$, $F_i(Q_t)$ and ν_{it}, i=1,2. For this case ν_{1t} will be

$$\nu_{1t} = \frac{\sigma_1^2 \bar{f}_{1t} f_{2t} + (Q_t^{(\lambda_1)} - x_{1t}'\beta_1) f_{1t} F_{2t}}{F_{1t} f_{2t} + f_{12} F_{2t}},$$

with a corresponding expression from ν_{2t}. We can also show that $E(\nu_{it}) = 0$, and clearly the ν_{it} are independent under I.

Under the above framework we can easily introduce $\bar{H}$, and test for $\bar{I}$ and $\bar{H}$ given $\bar{N}$ or test for $\bar{I}$, $\bar{N}$ and $\bar{H}$ simultaneously. One way to incorporate $\bar{H}$ would be to assume that the p.d.f. of $D^{(\lambda_1)}$ is $\phi_T(D^{(\lambda_1)}; X_1\beta_1, \Omega_1)$ where $\Omega_1 = \Sigma_1^{1/2} R_1(\psi_1) \Sigma_1^{1/2}$ where $\Sigma_1 = \text{diag}(\sigma_1^2 + z_t'\alpha_1)$ with z_t and α_1 being known and unknown vectors. Similarly, we can incorporate $\bar{H}$ in the distribution of $S^{(\lambda_2)}$. Under this general set-up, for a simultaneous test the null hypothesis would be H_o: $\lambda_i = 1$, $\alpha_i = 0$ and $\psi_i = 0$, i=1,2. Following the above lines, the LM test statistic can be easily derived and this can be viewed as a general model specification test. However, if we are interested in a

simpler overall model specification test without any specific class of alternatives in mind, it seems a wiser approach would be to use the White (1982) information matrix test. In the next subsection we outline the test procedure.

3.2 A General Model Specification Text

Let us start with the model (2.1)-(2.3) under the assumption that the u_{it} are iid $N(0,\sigma_i^2)$, i=1,2. We are interested in testing the validity of this model. Let us denote $\theta' = (\beta_1', \sigma_1^2, \beta_2', \sigma_2^2)$ and let the dimension of θ be p. Under this set-up the density function for the t-th observation is given by [see equation (2.10)]

$$g(Q_t;\theta) = F_{1t}f_{2t} + f_{1t}F_{2t} \cdot$$

If the model is correctly specified then

$$E_g\left[-\frac{\partial^2 \log g(Q;\theta)}{\partial\theta\partial\theta'}\right] = E_g\left[\left(\frac{\partial \log g(Q;\theta)}{\partial\theta}\right)\left(\frac{\partial \log g(Q;\theta)}{\partial\theta}\right)'\right],$$

where E_g denotes the expectation assuming $g(\cdot,\cdot)$ is the true probability density function. White's information matrix test essentially tests the above identity. Such a test can be based on the following quantities [see White (1982, p. 9)]

$$W_{t\ell} = \frac{\partial^2 \log g(Q_t;\hat{\theta})}{\partial\theta_j\partial\theta_k} + \frac{\partial \log g(Q_t;\hat{\theta})}{\partial\theta_j} \cdot \frac{\partial \log g(Q_t;\hat{\theta})}{\partial\theta_k}, \qquad (3.6)$$

$$j \leq k = 1, 2, \ldots, p, \quad \ell = 1, 2, \ldots, p(p+1)/2,$$

where $\hat{\theta}$ and θ_j are respectively the MLE and j-th element of θ. Asymptotic moments of the quantities in (3.4) under the null hypothesis are quite complicated. However, Chesher (1983) provides a simpler way to calculate an asymptotically equivalent version of the test statistic. Let W be a Txq matrix as defined in (3.4) with q = p(p+1)/2. Then the test statistic can be calculated as

$$S = \underline{1}' W(W'W)^{-1}W'\underline{1} = T \cdot R^2_{\underline{1} \cdot W},$$

where $\underline{1}$ is a Tx1 vector of ones and $R^2_{\underline{1} \cdot W}$ is the uncentered coefficient of detemination from the regression of a vector of ones on W. Under the null hypothesis, S is asymptotically distributed as χ^2 with q degrees of freedom. To calculate S all we need are the first and second derivatives of $g(Q_t;\theta)$ evaluated at $\theta = \hat{\theta}$, and these are given in Maddala and Nelson (1974, pp. 1016-1018). Therefore, it is not difficult to carry out this test. Here we should mention two points. First, the χ^2 degrees of freedom will be quite large even for a small model so the test may not be very powerful. One possible solution is to consider only elements with j=k in (3.6). Then we will have a χ^2 test with p degrees of freedom. Second, this is a pure significance test and therefore, once the null hypothesis of no misspecification is rejected, it does not provide any information regarding the direction of departure from the tested model. One possible strategy would be to apply one-directional tests such as the Lee (1982) non-normality test and a test for serial dependence separately with low significance levels.

4. Disequilibrium Analysis with Logistic Distribution

4.1 Preliminaries

One problem with the log-likelihood function based on the normal distribution is that its evaluation requires computation of normal integrals for which no closed form expressions are available. Also, earlier experience in estimating disequilibrium models has revealed difficulties in obtaining convergence using a Gauss-Newton type algorithm. Therefore, there is a need to look for alternative distributions which will lead to well-behaved and computationally simpler log-likelihood functions. In this section, we will explore the possibility of using the logistic distribution in analyzing disequilibrium models.

If a random variable z follows a univariate logistic distribution, its p.d.f h and distribution function H can be written as

$$h(z;\mu,\sigma) = \frac{1}{\sigma} \frac{\exp\{-(\frac{z-\mu}{\sigma})\}}{[1 + \exp\{-(\frac{z-\mu}{\sigma})\}]^2}$$

and

$$H(z;\mu,\sigma) = \frac{1}{1 + \exp\{-(\frac{z-\mu}{\sigma})\}}$$

$$-\infty < z < \infty, \; -\infty < \mu < \infty, \; 0 < \sigma < \infty,$$

respectively. It is well known that $E(z) = \mu$, $V(z) = \sigma^2 \frac{\pi^2}{3}$,

the coefficient of skewness = 0.0 and the coefficient of excess = 1.2. Therefore, the logistic distribution is very much like the normal but with a slightly thicker tail.

4.2 Log-likelihood Function and Estimation

If we assume that D_t and S_t follow two independent logistic distributions then the log-likelihood function for the disequilibrium model can be written as

$$L(\theta) = \sum_{t=1}^{T} \log G_t,$$

where

$$G_t = \frac{1}{\sigma_1} \frac{\exp\{-(\frac{Q_t - x_{1t}'\beta_1}{\sigma_1})\}}{[1 + \exp\{-(\frac{Q_t - x_{1t}'\beta_1}{\sigma_1})\}]^2} \cdot \frac{\exp\{-(\frac{Q_t - x_{2t}'\beta_2}{\sigma_2})\}}{[1 + \exp\{-(\frac{Q_t - x_{2t}'\beta_2}{\sigma_2})\}]}$$

$$+ \frac{\exp\{-(\frac{Q_t - x_{1t}'\beta_1}{\sigma_1})\}}{[1 + \exp\{-(\frac{Q_t - x_{1t}'\beta_1}{\sigma_1})\}]} \cdot \frac{1}{\sigma_2} \frac{\exp\{-(\frac{Q_t - x_{2t}'\beta_2}{\sigma_2})\}}{[1 + \exp\{-(\frac{Q_t - x_{2t}'\beta_2}{\sigma_2})\}]^2}.$$

By denoting $h_{it} = \frac{Q_t - x_{it}'\beta_i}{\sigma_i}$, $a_{it} = \exp(-h_{it})$, $i = 1, 2$, we have

$$L(\theta) = \sum_{t=1}^{T} [\log \sum_{i=1}^{2} \{\sigma_i(1+a_{it})\} - \sum_{i=1}^{2} \{h_{it} + 2\log(1+a_{it}) + \log \sigma_i\}] \quad (4.1)$$

The above log-likelihood function is much simpler than (2.10), or the expression given in Goldfeld and Quandt (1981, p. 149) based on the Sargan density. It is clear that $L(\theta)$ given in (4.1) is continuously differentiable. By choosing appropriate parameter values such as $\sigma_1 = 0$, $x_{11}\beta_1' = Q_1$, $\sigma_2 \neq 0$, $x_{1t}\beta_1' \neq Q_t$ for $t \neq 1$, it is seen that $L(\theta)$ is unbounded. This property also holds for the log-likelihood function based on the normal distribution [see Quandt (1982, p. 13)].

It would be interesting to investigate the performance of the two log-likelihoods given in (2.10) and (4.1) in terms of computation and statistical properties of the estimates when the data are generated from different distributions. This is what we set out to do in the next section through simulation experiment.

4.3 Simulation Results

The simulation experiment was carried out using the housing start model given in (2.16)-(2.18) and the Fair and Jaffee data set on Z variables. The disturbance terms u_1 and u_2 were generated as the normal, logistic and Student's t with 7 degrees of freedom. For the population parameter values we used the MLEs of Maddala and Nelson (1974) except we set $\alpha_0 = 171.8$, $\sigma_1^2 = 40$ and $\sigma_2^2 = 30$. These changes were made to make the task of maximizing the log-likelihood function a little easier. Observations on D_t and S_t were obtained using (2.16) and (2.17), and the (2.18) provided the Q_t values. We generated 100 samples each of size 126 under the above three distributions and estimated the model assuming that the underlying distribution is either

normal or logistic. We employed a Gauss-Newton type iteration scheme to obtain the MLEs, namely,

$$\theta_{(r)} = \theta_{(r-1)} + \delta(A'A)^{-1}A'\underline{1}\Big|_{(r-1)}, \quad r \geq 2,$$

where $\theta_{(r)}$ is the value of θ at the r-th iteration, δ is a scalar chosen at each iteration step and $A = ((A_{tj})) = ((\partial L_t(\theta)/\partial\theta_j))$, $L_t(\theta)$ being the log-density function for the t-th observation. We experimented with many starting values $\theta_{(1)}$ for the iterations, such as estimates obtained from the Directional Method I considered by Fair and Jaffee (1972) and final iterated values from an arbitrary replication. (In many cases, the iterations converged to different points which were close to the starting values indicating the presence of locally optimal points). After much trial and error, we found that the latter option gave better results. For the logistic log-likelihood we used numerical derivatives, whereas for the normal log-likelihood we had to provide the analytical derivatives in order for the iteration process to converge. This indicates one advantage of using the logistic distribution.

After obtaining the estimates from 100 replications we calculated the root mean square errors (RMSE's) for different cases, and these are reported in Table 4.1. The results are quite mixed and the RMSE's do not differ much for the six cases. Use of the correct distribution does not always lead to lower RMSEs. This may be, apart from convergence to suboptimal points, due to finite-sample errors and/or the small number of replications. Under the correct setup, the logistic distribution does slightly better than the normal, which is

Table 4.1

Comparison of Root Mean Square Errors

True Distribution / Assumed Distribution		Normal		Logistic		Student's t	
		Normal	Logistic	Normal	Logistic	Normal	Logistic
Parameters	α_0	10.6150	9.5568	10.6073	10.2584	10.5496	11.3700
	α_1	.1532	.1382	.1531	.1487	.1523	.1654
	α_2	.0014	.0013	.0014	.0014	.0014	.0015
	α_3	.1750	.1768	.1750	.1770	.1751	.1774
	β_0	.9774	.8912	.9768	.9613	.9715	1.0711
	β_1	.7816	.6293	.7804	.6767	.7756	.6634
	β_2	.2860	.2224	.2856	.2217	.2839	.2186
	β_3	.2558	.1950	.2553	.1951	.2536	.1941
	β_4	.3521	.3419	.3507	.3503	.3481	.4123

not the case when we use the normal distribution correctly. When the distribution is misspecified, the performance of the normal distribution is better, though the differences are not substantial. Therefore, bearing in mind its computational advantages, the logistic distribution might be preferred to the normal in the analysis of market disequilibrium models with independent disturbances.

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